Bode Plots

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A general transfer function can be represented in the following way:

$$H(s) = k\frac{\prod_{i=1}^{n_1} 1+\alpha_i s \prod_{i=1}^{n_2} \left(\frac{s}{\Omega_i}\right)^2 + \frac{2\xi_i s}{\Omega_i} + 1}{\prod_{i=1}^{n_3} 1+\beta_i s \prod_{i=1}^{n_4} \left(\frac{s}{\omega_i}\right)^2 + \frac{2\zeta_i s}{\omega_i} + 1}$$

Bode plot is a plot of $20\log_{10}|H(\jmath\omega)|$ and the $\angle H(\jmath\omega)$ versus ω. It is a semilog plot where the x-axis is the log-axis and the y-axis is the linear axis. Since $\log(ab) = \log a + \log b$ and also phase of the product of two complex numbers is the sum of their phases we can plot the Bode plot of each factor separately and then just add them up. For this reason we look at the Bode plot of each factor separately.

Second Order Network

$$H_1(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{2\zeta s}{\omega_0} + 1}$$

$$H_1(\jmath\omega) = \frac{1}{\left(\frac{\jmath\omega}{\omega_0}\right)^2 + \frac{2\zeta\jmath\omega}{\omega_0} + 1} = \frac{1}{1-\left(\frac{\omega}{\omega_0}\right)^2 + \frac{2\zeta\jmath\omega}{\omega_0}}$$

$$|H_1(\jmath\omega)| = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 - 4\left(\frac{\zeta\omega}{\omega_0}\right)^2}}$$

Define $u \triangleq \frac{\omega}{\omega_0}$ and now the above expression can be written as:

$$|H_1(\jmath u)| = \frac{1}{\sqrt{\left(1-(u)^2\right)^2 - 4(\zeta u)^2}} = \frac{1}{\sqrt{1+2u^2(2\zeta^2-1)+u^4}}$$

$$\frac{d|H_1(\jmath u)|}{du} = -\frac{1}{2}\frac{4u^3+4u(2\zeta^2-1)}{(1+2u^2(2\zeta^2-1)+u^4)^{\frac{3}{2}}}$$

$$\frac{d|H_1(\jmath u)|}{du} = 0 \text{ when } u=0 \text{ or } u=\sqrt{1-2\zeta^2}, \text{ for u real } \zeta < \frac{1}{\sqrt{2}}$$

$$\left|H_1(\jmath\sqrt{1-2\zeta^2})\right| = \frac{1}{\sqrt{1-(1-2\zeta^2)^2}}$$